Chapter 16 – RANDOM VARIABLES

Vocabulary

Random Variable – A random variable assumes any of the several different values as a result of some random event. Random variables are denoted by a capital letter, like X.

Discrete Radom Variable – A random variable that can take one of a finite number of distinct outcomes.

Continuous Random Variables – A random variable that can take any numeric value within a range of values. The range may be infinite or bounded at either at both ends.

Probability Model – A function that associates a probability P with each value of a discrete random variable X, denoted P(X=x) or with any interval of values of a continuous random variable.

⇒ We use probability models to demonstrate what we expect will happen.

Expected Value – The expected value of a random variable is the theoretical mean/average, the center of the model. Denoted μ or E(X), it is found (if the random variable is discrete) by adding the products of variable values and probabilities.

⇒ $\mu = E(X) = \sum x * P(x)$

Standard Deviation – The standard deviation of a random variable describes the spread in the model, and is the square root of the variance.

⇒ $\sigma = SD(x) = \sqrt{var(X)}$

Variance – Expected value of squared deviation from mean. The sum of two independent random variables is the sum of their individual variances.

⇒ $\sigma^2 = var(X) = \sum (x-\mu)^2 P(x)$

**In general, multiplying each value of a random variable by a constant multiplies the mean by that constant and the variance by the SQUARE of the constant.*
**The mean of the sum of two random variables is the sum of means*
**The mean difference of two random variables is the difference of the means*
**If the random variables are independent, the variance of their sum or difference is always the sum of the variances.*

E(X +- Y) = E(X) +- E(Y)   Var(X +- Y) = Var(X) +- Var(Y)

**"Probability models are just models" – Models can be useful, but they are NOT reality.*
**Don't assume that everything resembles the Normal Model*

Examples

1) What is the probability that packing two consecutive systems takes over 20 minutes? (See prompt on pg. 376)

√Normal Condition: We are told that both random variables follow the Normal Model
√Independence: We can assume that the two packing times are independent of one another
$T=P_1 + P_2$ P_1= time for packing 1st system P_2 = time packing for 2nd system

$E(T) = E(P_1 + P_2) = E(P_1) + E(P_2) = 9+9 = 18$ minutes

Since the times are independent $Var(T) = Var(P_1 + P_2) = 1.5^2 + 1.5^2$

$Var(T) = 4.50$ $SD(T) = 4.50^{1/2} = 2.12$ minutes

$Z = (20-18)/2.12 = 0.94$

$P(T>20) = P(z>0.94) = 0.1736$

∴ There is slightly above a 17% chance that it will take over 20 minutes to pack two consecutive stereo systems.

2) What percent of stereo systems take longer to pack than to box?

√Normal Condition: We are told tat both random variables follow the Normal Model
√Independence: We can assume that the time it takes to pack and to box a system are independent

E(Difference) = E(Packing-Boxing) = E(P) – E(B) = 9-6 = 3 minutes

Since the times are independent $Var(D) = Var(P-B) = Var(P) – Var(B) = 1.5^2 + 1^2$

$Var(D) = 3.25$ $SD(D) = 3.251/2 = 1.80$ minutes

$Z = (0-3)/1.80 = -1.67$

$P(D>0) = P(z>-1.67) = 0.9525$

∴ About 95% of all the stereo systems will require more time for packing than for boxing.